Finite Element Method

Homework II
Due date: April 20.

1.

(i) Find the energy functional of the Bernulli-Euler beam and derive the Euler Lagrange equation for the Bernulli-Euler beam.

(ii) What are possible boundary conditions that can ensure a unique solution when uniform loads are applied to the beam? Justify your answer and explain the physical meaning of each boundary condition in yours answers. Recall that the state variables are w and θ, where $\theta = -\frac{\partial w}{\partial x}$, and the bending moment is defined by $M = EI\frac{\partial^2 w}{\partial x^2}$.

(iii) Prove that the solution of the Euler Lagrange equation is indeed the global minimizer of the energy functional.

2. Let the functional $J[y] = \int_{-1}^{1} x^{2/3}(y')^2\,dx$, where $y(-1) = -1$ and $y(1) = 1$.

(i) Find the corresponding Euler-Lagrange equation.

(ii) Find the minimum of the functional J and prove your answer is correct.

(iii) Show that your answer is indeed a function in $H^1([-1,1])$.